THE HELMHOLTZ-KELVIN TIME SCALE FOR STARS OF VERY LOW MASS

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ABSTRACT

Assuming that the contracting stars in convective equilibrium evolve vertically downward in the H-R diagram, a simple expression for the Helmholtz-Kelvin time scale t_{HK} is derived. Application of this expression to stars of mass $M < 0.1M_{\odot}$ shows that these stars contract to a radius of about $0.1R_{\odot}$ in a time scale of approximately 1 billion years, while the earlier estimates, based on horizontal evolution, gave a time scale t_{HK} greater than a hundred billion years.

I. INTRODUCTION

Until 1961, it was believed that the time scale for the Helmholtz-Kelvin contraction of stars of low mass ($M < 0.1M_{\odot}$) was greater than a hundred billion years. This belief was based on the assumption that the contracting stars evolved horizontally in the H-R diagram with a very low luminosity. A constant luminosity of the order of $10^{-6}\,L_{\odot}$ gave a very long time scale for contraction down to a radius of about $0.1R_{\odot}$. However, recent work by Hayashi (1962) has shown that the contracting stars of low mass remain completely convective during the premain-sequence contraction. The completely convective nature of these stars makes them much more luminous than the models based on the assumption of complete radiative equilibrium (Levee 1953). Consequently, the completely convective contracting stars evolve much more rapidly. Following this suggestion by Hayashi, we shall compute here the time scale t_{HK} for stars of mass $M < 0.1M_{\odot}$. Before we do that, we shall derive a simple expression for the time scale for contraction from a radius R_1 to R_2.

II. EXPRESSION FOR THE TIME SCALE

Hayashi's work and recent work by Ezer and Cameron (1962) and Kumar and Upton (1963) show that the completely convective stars contract in such a way that their evolutionary path in the H-R diagram is nearly a vertical line. We shall make use of this vertical evolution to obtain an expression which gives the time scale t_{HK} quite accurately. The luminosity of a contracting star is given by

$$L = -\frac{\Delta E}{\Delta t} = -\frac{3\gamma - 4}{3(\gamma - 1)}\frac{d\Omega}{dt} = 4\pi R^2 \sigma T_e^4\,, \tag{1}$$

where γ is the ratio of the two specific heats c_p and c_v and Ω is the potential energy of the star. For a completely convective star, which can be represented by a sphere of polytropic index $n = 1.5$, Ω is given by

$$\Omega = -q\frac{GM^2}{R}\,, \tag{2}$$

where

$$q = \frac{3}{5 - n} = \frac{6}{7}\,. \tag{3}$$

Here R is the radius of the star, T_e its effective temperature, and σ the Stefan-Boltzmann constant. From equations (1), (2), and (3) we obtain

$$dt = -\frac{3GM^2}{28\pi\sigma T_e^4}\frac{dR}{R^4}\,. \tag{4}$$

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In writing equation (4), we have put $\gamma = \frac{5}{3}$. We treat T_e as a constant, as the vertical evolution in the H-R diagram means that the star's effective temperature remains constant during contraction. Integrating equation (4), we obtain

$$t_{\mathrm{HK}} = \frac{GM^2}{28\pi\sigma T_e^4}\left(\frac{1}{R_2^3} - \frac{1}{R_1^3}\right). \tag{5}$$

If we take $R_1 = \infty$, the required expression for t_{HK} is

$$t_{\mathrm{HK}} = 4.98 \times 10^9 \frac{M^2}{T_3^4 R_2^3}, \tag{6}$$

where M and R_2 are in solar units, T_3 is the effective temperature in thousands of degrees, and t_{HK} is the time scale in years. For comparison, we give here the corresponding equation for contracting stars in radiative equilibrium (Levee 1953):

$$t_{\mathrm{HK}} = 2.51 \times 10^7 \frac{M^2}{\bar{L} R_2}, \tag{7}$$

where $\bar{L}$ is the mean luminosity during contraction.

In order to use equation (6), we must know R_2 and T_3 for a star of a given mass. Before we apply it to stars of low mass, let us apply it to the contracting sun while it remains a fully convective star. Ezer and Cameron (1962) have found that it remains fully convective up to a radius of 3.0 and that the time scale up to this stage is 4.6×10^5 years. The mean effective temperature during this contraction is 4400° K, and so from equation (6) we have

$$t_{\mathrm{HK}} = 5.0 \times 10^5 \text{ years}. \tag{8}$$

This agreement between the two time scales shows that equation (6) is fairly accurate.

III. APPLICATION TO STARS OF LOW MASS

To compute t_{HK} for stars of low mass, we choose three masses: 0.09, 0.07, 0.05. For each mass we consider the contraction down to the radius R_2 at which the central temperature reaches a maximum value. At this stage a population I star ($X = 0.62$, $Y = 0.35$, $Z = 0.03$) of mass 0.09 most probably becomes a main-sequence star, while the less massive stars begin to cool toward complete degeneracy (Kumar 1963). For R_2 we use the radii corresponding to the stage of maximum central temperature computed by Kumar. As far as T_3 is concerned, we use a value that is substantially lower than the range in which the actual temperature is likely to lie. By doing this, we should get t_{HK} greater than the actual time scale.

The choice of T_3 for these masses is based on the known effective temperature for Ross 614B and L726–8. For Ross 614B, which was at one time a contracting star—or it may still be contracting slowly—Limber (1958) gives $M = 0.0766$, $R = 0.0955$, and $T_3 = 2.7$. If we put these values in equation (6), we obtain $t_{\mathrm{HK}} = 6 \times 10^8$ years. For L726–8, van de Kamp (1959) gives the total mass of 0.08 for the system. Further, he has shown that both components are equally massive and therefore each of them has a mass of 0.04. The spectral type of the brighter component is dM6e, and this corresponds to a temperature of 2750° K. The values of T_3 given in Table 1 have been chosen, keeping in mind the temperatures for these two stars.

Table 1 gives R_2, T_3, and t_{HK} for each of the three masses. Our computations show clearly that the stars of low mass contract down to a small radius in a time scale which is much smaller than the earlier estimate for the quantity t_{HK}. In particular, the new time scales computed here are small as compared with the age of the Galaxy. Therefore, we may say that the number of stars of low mass in the Galaxy that have evolved beyond the stage of maximum central temperature is very large, although their contribution to the total mass of the Galaxy may not be significant.

It will be noticed that the time scale t_{HK} for the star of mass 0.05 is smaller than for that for a star of mass 0.09. This is easily explained by the effects of degeneracy. Because the degeneracy sets in at a larger radius in the less massive star, the stage of maximum central temperature is reached at a larger radius. Thus the time t_{HK}, which is inversely proportional to the cube of the radius, is smaller for a star of mass 0.05 than that for a star of mass 0.09. If we consider a mass slightly higher than 0.09, the time scale decreases. Kumar and Upton (1963) have found that, for a star of mass 0.1, the time scale for contraction down to the main-sequence stage is approximately 3×10^8 years. Therefore, we may say that, at $M = 0.09$, the time scale t_{HK} has a maximum value.

So far, we have neglected the effect of the nuclear reactions involving the destruction of deuterium, lithium, beryllium, and boron in the contracting stars. We would expect these reactions to slow down the contraction, and thus the time scales computed above would have to be increased. Salpeter (1955) has computed the reaction rates for the destruction of H^2, Li^6, Li^7, Be^9, B^{10}, and B^{11}. Making use of Salpeter's results and

TABLE 1

TIME SCALES FOR THREE MASSES

M	R_2	T_2	t_{HK}
0.09	0.11	2.5	8×10^8
.07	.12	2.2	5×10^8
0 05	0.135	1.9	4×10^8

the central temperatures and densities computed by Kumar (1963), we see that deuterium will burn in these stars in a time scale much smaller than 1 billion years. Thus the destruction of H^2 will not increase appreciably the time scales t_{HK} given in Table 1. The same argument holds for the reactions involving Li^6 and Li^7. In a star of mass 0.09, the destruction of Be^9 requires a short time, but in stars of mass 0.07 or 0.05 it may not burn at all. Similarly B^{11} may be destroyed in a short time in a star of mass 0.09, but it will not burn in a star of mass 0.07 or 0.05. It seems that B^{10} will not burn in any of these three masses. Thus the destruction of the elements H^2, Li^6, Li^7, Be^9, B^{10}, and B^{11} does not increase appreciably the time scales given in Table 1. These time scales may be increased by a factor of, at most, 2 because of the destruction of these elements.

We conclude that the time scale t_{HK} for contraction to a radius of about $0.1R_{\odot}$ for stars of low mass ($M < 0.1$) is approximately 1 billion years and not a hundred billion years or more, as was believed to be 1 year ago.

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